

Chiral specific electron vortex beam spectroscopy

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Electron vortex beams carry well-defined orbital angular momentum (OAM) about the propagation axis. Such beams are thus characterised by chirality features which make them potentially useful as probes of magnetic and other chiral materials. An analysis of the inelastic processes in which electron vortex beams interact with atoms and which involve OAM exchange is outlined, leading to the multipolar selection rules governing this chiral specific electron vortex beam spectroscopy. Our results show clearly that the selection rules are dependent on the dynamical state and location of the atoms involved. In the most favorable scenario, this form of electron spectroscopy can induce magnetic sublevel transitions which are commonly probed using circularly polarized photon beams.

Particle vortices, most notably electron vortices (EVs), are currently the focus of much interest following the prediction of their existence on a theoretical basis by Bliokh *et al.* [1]. Such vortices have been created in a number of laboratories, using various techniques. [2–7]. This area has recently emerged after much fruitful research has been carried out on optical vortices (OVs) over the last two decades or so, which has led to a wealth of fundamental knowledge and significant applications [8–10]. Both the optical and electron vortices are characterised by the singular nature of their wavefronts, with a well defined vortex core and quantised orbital angular momentum (OAM) about the vortex axis. The general expectation is that in all cases the vortex OAM should play an important role in the interaction of the vortex with matter. However, in the case of an OV, a dipole active transition involves exchange of OAM with the centre of mass only [11, 12], a finding which has been confirmed experimentally [13, 14]. The transfer of the OAM of an OV to the centre of mass of bulk matter has been demonstrated, most clearly in the optical spanner effect [15, 16], but as far as optical spectroscopy is concerned the development of OAM-based OV beam spectroscopy has been hampered by the weakness of optical multipolar transitions. In contrast, we have recently shown that OAM can be transferred efficiently from an EV beam to atomic electrons through dipole active transitions [12, 17] and experimentally, a dichroic electron energy loss spectroscopy signal has been demonstrated [3], opening up the prospect of chiral specific electron vortex beam spectroscopy (CEVBS) based on OAM selection rules. This article seeks to determine the key characteristics of OAM- and chiral-related EV beam spectroscopy by analysing the OAM selection rules, and specifying the spatial dependence of the atomic transitions involved.

The leading interaction between the EV and an atom possessing Z electrons is given by the Coulomb interaction Hamiltonian

$$\hat{H}_{\text{int}} = -\frac{Ze^2}{4\pi\epsilon_0|\mathbf{r}_v - \mathbf{R}|} + \sum_{j=1}^Z \frac{e^2}{4\pi\epsilon_0|\mathbf{r}_v - \mathbf{r}_j|}, \quad (1)$$

where \mathbf{r}_v , \mathbf{r}_j and \mathbf{R} are the position vectors, respectively,

of the beam electron, the j -th atomic electron and the nucleus, all expressed relative to the laboratory frame of reference. The transition matrix element between overall eigenstates of the atom and vortex system can be written as $M_{fi} = \langle F | \hat{H}_{\text{int}} | I \rangle$, where $|I\rangle$ and $|F\rangle$ are, respectively, the initial and final unperturbed quantum states of the overall system, being products of unperturbed quantum states of the EV and the atom: $|\psi_{\text{EV}}\rangle |\psi_{\text{atom}}\rangle$. This is justified as the energy of the EV beams investigated so far (100kV or more) is much larger than the energy of the atomic electrons and that of the atomic centre of mass. The interesting case in which a low energy vortex beam interacts with the atom requires a different treatment involving particle exchange, and is not considered any further here. In the present case the atomic quantum state can be taken as a product of the quantum state of its nucleus, here taken to also be the centre of mass of the atom, and that describing the internal electronic state relative to the centre of mass \mathbf{R} , i.e. $|\psi_{\text{atom}}\rangle = |\psi_{\text{cm}}(\mathbf{R})\rangle |\psi_q(\mathbf{r}_1, \dots, \mathbf{r}_j, \dots, \mathbf{r}_Z)\rangle$.

Without loss of generality, we assume a Bessel EV beam of winding number l which is an eigenstate of the Schrödinger equation for a free electron in cylindrical polar coordinates (ρ_v, ϕ_v, z_v) with the beam axis along the z -direction,

$$|\psi_{\text{EV}}\rangle = |k_{\perp}, l, k_z\rangle_{\text{lab}} = \frac{\sqrt{k_{\perp}}}{2\pi} J_l(k_{\perp}\rho_v) e^{il\phi_v + ik_z z_v + i\omega t}, \quad (2)$$

where k_{\perp} and k_z are the transverse and longitudinal components of the wavevector of the vortex beam such that $k_{\perp}^2 + k_z^2 = k^2 = \frac{2mE}{\hbar^2}$, with E the beam energy, and $J_l(k_{\perp}\rho_v)$ is the l th order Bessel function satisfying the orthonormalisation condition

$$\int_0^{\infty} J_l(k_{\perp}\rho_v) J_{l'}(k'_{\perp}\rho_v) \rho_v d\rho_v = \frac{1}{k_{\perp}} \delta_{l,l'} \delta(k_{\perp} - k'_{\perp}), \quad (3)$$

which, along with the usual delta function normalisation over z gives the normalisation factor of Eq. (2).

CEVBS is concerned with processes in which an incident EV mode $|k_{\perp}, l, k_z\rangle_{\text{lab}}$ is scattered by the atom into an outgoing EV mode $|k'_{\perp}, l', k'_z\rangle_{\text{lab}}$, with the atom un-

dergoing a quantum transition between its internal eigenstates. The treatment can be readily extended to more general vortex beams since such beams can be represented by a linear combination of the Bessel basis modes discussed here. As a simplification which is justifiable for spectroscopy of condensed matter systems, we shall initially assume that the scattering process does not alter the state of the atomic centre of mass, which we take as stationary. Although this fixed centre of mass assumption is adequate in the present context of chiral specific spectroscopy, the centre of mass coordinate must be retained as a dynamical variable, as done in the case of OV's [10, 18, 19] when the gross atomic motion is of interest due to the influence of forces and torques imposed by the EV on the atom.

As in the case of the optical vortex interacting with the atom, the transition matrix element for an EV interacting with the atom may be reduced to the following form [20]

$$\mathcal{M}_{fi} = \frac{e^2}{4\pi\epsilon_0} \sum_j \langle f | \hat{O}_j^{l,l'} | i \rangle, \quad (4)$$

where $|i\rangle$ and $|f\rangle$ are the initial and final states of the atom. The effective operator $\hat{O}_j^{l,l'}$ acts on a single electron, and emerges in the form

$$\hat{O}_j^{l,l'} = \frac{\sqrt{k_\perp k'_\perp}}{4\pi^2} \int_{-\infty}^{\infty} \frac{F_v^{l,l'} e^{i(k_z - k'_z)z_v}}{|\mathbf{r}_v - \mathbf{r}_j|} d^3 r_v, \quad (5)$$

where

$$F_v^{l,l'} = J_l(k_\perp \rho_v) J_{l'}(k'_\perp \rho_v) e^{i(l-l')\phi_v}. \quad (6)$$

Note that the first term in Eq. (1) does not contribute to the matrix element by virtue of the orthogonality of the initial and final atomic states $|i\rangle$ and $|f\rangle$.

The chief difficulty in the evaluation of the effective operator for the vortex beam-atom interaction in Eq. (5) stems from the fact that the vortex state function is conveniently expressed in terms of the laboratory frame, while the internal atomic states are customarily expressed in spherical coordinates in a frame of reference centred on the atomic centre-of-mass of coordinate \mathbf{R} . In order to overcome this difficulty, the addition theorem of Bessel functions [21, 22] can be utilised to represent the original EV beam of mode l as a sum of other vortex states relative to a shifted frame of reference centred on the atomic centre of mass coordinate \mathbf{R} . The addition theorem reads

$$J_\mu(a) = e^{-i\mu\theta} \sum_{\nu=-\infty}^{\infty} J_{\mu+\nu}(b) J_\nu(c) e^{i\nu\varphi}, \quad (7)$$

where a , b and c are three sides of a triangle, and θ and φ the internal angles between sides a and b and b and c respectively. Applying this to the triangle formed by

the position vectors of the vortex, nucleus and atomic electron, we identify $\mathbf{r}_c(\rho_c, \phi_c, z_c)$ as the position vector describing the vortex electron relative to the centre of mass, and after some further algebraic manipulation, we find

$$J_l(k_\perp \rho_v) = e^{-il\phi_v} \sum_{p=-\infty}^{\infty} J_{l-p}(k_\perp \rho_R) J_p(k_\perp \rho_c) e^{i(l-p)\phi_R} e^{ip\phi_c}. \quad (8)$$

The EV beam may now be written as the sum of vortex modes relative to the centre of mass

$$|k_\perp, l, k_z\rangle_{\text{lab}} = \sum_{p=-\infty}^{\infty} C_{l-p} |k_\perp, p, k_z\rangle_{\text{cm}}, \quad (9)$$

where the subscripts refer to the reference frame relative to which the quantum states are described, and the coefficient C_{l-p} is the overlap integral, given by

$$C_{l-p} = J_{l-p}(k_\perp \rho_R) e^{i(l-p)\phi_R}. \quad (10)$$

As expected, the above expansion indicates that the only vortex mode relative to the centre of mass present for an atom located on the beam axis is that for which $p = l$. However, for an atom not situated on the beam axis, the strength of the atom-centred vortex modes with $p = l + 1$ and $p = l - 1$ also become significant when the atom is positioned at radial distances of the order of a fraction of $\frac{\alpha_{l,1}}{k_\perp} \approx 0.1$ nm, where $\alpha_{l,1}$ is the first zero of the l th-order Bessel function, i.e. within the first ring of the vortex beam. Thus the immediate consequence of the shift of the axis is the importance of vortex modes of winding numbers different from l , relative to the atomic centre of mass frame, as is the case in the context of OV beams. This is a manifestation of the extrinsic property of the orbital angular momentum of the beam [23] - once we transform from the symmetry axis of the vortex to one centred on the atomic centre of mass \mathbf{R} the OAM also changes, as emphasised by Berry [24]

$$L_z \rightarrow L_z + \mathbf{e}_z \cdot \rho_R \wedge \mathbf{P}, \quad (11)$$

where \mathbf{P} is the transverse current in the vortex beam. Thus the OAM assumes different values for an off-axis atom at different positions relative to the symmetry axis.

Applying the shifted wavefunctions of Equations (9) and (10), the effective operator $\hat{O}^{l,l'}$ (the subscript j will henceforth be dropped) relative to the atomic frame can be written as follows

$$\hat{O}^{l,l'} = \frac{\sqrt{k_\perp k'_\perp}}{2\pi} e^{-i(k_z - k'_z)z_R} \sum_{p,p'=-\infty}^{\infty} F_R^{l-p,l'-p'} I_c^{p,p'}, \quad (12)$$

where

$$F_R^{l-p,l'-p'} = J_{l-p}(k_\perp \rho_R) J_{l'-p'}(k'_\perp \rho_R) e^{-i(p-p'-l+l')\phi_R}, \quad (13)$$

and

$$I_c^{p,p'} = \int \frac{F_c^{p,p'} e^{i(k_z - k'_z)z_c}}{|\mathbf{r}_q - \mathbf{r}_c|} d^3 r_c, \quad (14)$$

with $\mathbf{r}_q(\rho_q, \theta_q, \phi_q) = \mathbf{r}_j - \mathbf{R}$ and $\mathbf{r}_c(\rho_c, \theta_c, \phi_c) = \mathbf{r}_v - \mathbf{R}$, being the internal electronic and vortex beam coordinates respectively, both described about the atomic centre, and

$$F_c^{p,p'} = J_p(k_\perp \rho_c) J_{p'}(k'_\perp \rho_c) e^{i(p-p')\phi_c}. \quad (15)$$

We have also isolated the centre-of-mass factor F_R , relative to the atomic frame, from the integral I_c involving the atomic states $|i\rangle$ and $|f\rangle$.

In order to evaluate the matrix element in terms of multipole excitations, the effective operator must be expanded in powers of \mathbf{r}_q . This can be achieved by invoking the addition theorem for Bessel functions again in order to achieve a separation of the dependence on the atomic electron position variable \mathbf{r}_q from that of the EV \mathbf{r}_c . This is conveniently done by introducing a relative position vector $\mathbf{s} = (\mathbf{r}_c - \mathbf{r}_q)$. After some algebraic manipulation, the integral can be written as follows

$$I_c^{p,p'} = \sum_{u,u'=-\infty}^{\infty} F_q^{p-u,p'-u'} I_s^{u,u'}, \quad (16)$$

where F_q is given by

$$F_q^{p-u,p'-u'} = J_{p-u}(k_\perp \rho_q) J_{p'-u'}(k'_\perp \rho_q) e^{-i(u-u'-p+p')\phi_q}, \quad (17)$$

and $I_s^{u,u'}$ is given by

$$I_s^{u,u'} = \int d^3 r_s \frac{J_u(k_\perp \rho_s) J_{u'}(k'_\perp \rho_s) e^{i(u-u')\phi_s} e^{i(k_z - k'_z)z_s}}{(\rho_s^2 + z_s^2)^{1/2}}. \quad (18)$$

The integral $I_s^{u,u'}$ can be evaluated by expressing each Bessel function as a coherent superposition of plane waves with phase angle dependent on the topological charge [21]. The linear momentum transfer wavevector is defined as $\mathbf{Q}(\beta) = \mathbf{k}_f - \mathbf{k}_i$, where \mathbf{k}_i is the wavevector of the plane-wave components of the incident Bessel beam, and \mathbf{k}_f is that of the outgoing Bessel beam, with $\beta = \phi - \phi'$ the relative azimuthal angle between \mathbf{k}_i and \mathbf{k}_f . The integral over the vortex beam spatial variables may now be evaluated as the Fourier transform of the Coulomb potential, leading to the condition $u = u'$:

$$I_s^u = \frac{1}{\sqrt{2\pi^3}} \int_0^{2\pi} \frac{e^{iu\beta}}{Q^2(\beta)} d\beta. \quad (19)$$

We note that $1/Q^2(\beta)$ is the familiar kinetic factor arising in Coulomb scattering.

The result for the effective operator of the vortex electron beam can now be determined by combining Equations (12), (13), (16), (17) and (19):

$$\hat{O}^{l,l'} = \hat{O}^z \frac{\sqrt{k_\perp k'_\perp}}{4\pi^2} \sum_{p,p'=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} F_R^{l-p,l'-p'} F_q^{p-u,p'-u} I_s^u, \quad (20)$$

where $\hat{O}^z = e^{i(k_z - k'_z)(z_R + z_q)}$ is the effective operator for out-of-plane excitations. Eq. (20) allows a clear description of the effect of the EV beam expansion - contained in the F_R factors - and its implications for the OAM transfer between the EV and the atomic electron - contained in the F_q factors. We illustrate this by considering the implications for chiral specific vortex electron beam spectroscopy.

Since the effective operator $\hat{O}^{l,l'}$ acts on the electronic states of the atoms only through terms containing components of \mathbf{r}_q , only the terms involving \hat{O}^z and F_q are relevant. Focusing on \hat{O}^z first, we can write $e^{i(k_z - k'_z)z_q}$ as $1 - i(k_z - k'_z)z_q + \mathcal{O}(z^2)$. Thus, the out-of-plane excitations importantly have no chirality features.

On the other hand, the term $F_q^{p-u,p'-u}$ depends on the in-plane components of \mathbf{r}_q , and contains the phase factor $e^{i(p-p')\phi_q}$ which is important for chiral specific spectroscopy. This becomes clear if we consider the simplest case of an atom located on the beam axis, in which case $\mathbf{R} = \mathbf{0}$. We see that F_R is non-zero only for $p = l$ and $p' = l'$, so that the summation over p, p' in Eq. (20) amounts only to a single term with $p = l$ and $p' = l'$, and $F_R = 1$. The simplified operator can then be written, in ascending powers of ρ_q ,

$$\begin{aligned} \hat{O}^{l,l'} = & \frac{\sqrt{k_\perp k'_\perp}}{2\pi} e^{-i(k_z - k'_z)z_q} \times \left[\mathcal{A}^0 \delta_{l,l'} \right. \\ & \left. + (\mathcal{A}^{+1} e^{i\phi_q} \delta_{l,l'+1} + \mathcal{A}^{-1} e^{-i\phi_q} \delta_{l,l'-1}) \rho_q + \mathcal{O}(\rho_q^2) \right] \end{aligned} \quad (21)$$

with

$$\begin{aligned} \mathcal{A}^0 &= I_s^l \\ \mathcal{A}^{+1} &= \frac{1}{2} (k_\perp I_s^{l-1} - k'_\perp I_s^l) \\ \mathcal{A}^{-1} &= \frac{1}{2} (-k_\perp I_s^{l+1} + k'_\perp I_s^l). \end{aligned}$$

In arriving at this result, we have expanded the Bessel functions involved in the asymptotic limit of small argument as [21]

$$J_n(k_\perp \rho_q) = (-1)^{-n} J_{-n}(k_\perp \rho_q) \approx \frac{1}{\Gamma(n+1)} \left(\frac{k_\perp \rho_q}{2} \right)^n, \quad (22)$$

where $\Gamma(z)$ is the Gamma function. This is equivalent to restricting EV spectroscopy to the limit in which the transverse momentum of the beam is small compared to its axial momentum, a condition often observed in high energy electron energy-loss spectroscopy [25]. In such cases, the dipole terms in Eq. (21) containing the factors $\rho_q e^{\pm i\phi_q} \delta_{l,l'\pm 1}$, when operating on the atomic state, causes the magnetic quantum number m of the electronic state to increase or reduce by one unit as a result of the transfer of one unit of OAM from the EV beam, leading to the selection rule $l - l' = -m + m' = \pm 1$. This is precisely the result obtained by Lloyd *et al.* [12, 17]. The

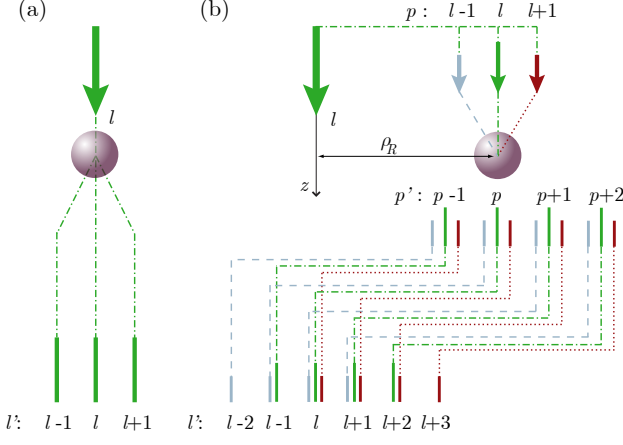


FIG. 1. (Colour Online) Transfer of OAM in an atom-vortex interaction in the cases when the atom is situated (a) on the beam axis, and (b) off axis. In (a) the resulting l' states relate directly to an interaction in which $l - l'$ units of OAM are exchanged; however it is not possible to determine, for example, whether an interaction for which $l - l' = 1$ is due to a dipole interaction, or a higher multipole. In (b), the final states l' arise due to a set of transitions in which varying quantities of OAM are exchanged with the different p -modes the atom “sees”. As an example, the $p = l - 1$ mode may be scattered to the $p' = p + 2$ mode - which may occur in a quadrupole or higher multipole interaction - resulting in a final state $l' = l + 1$. Thus the final states of the vortex beam are not an obvious indicator of the multipolar nature of the interaction. The strengths of the modes p and p' are dependent on several factors, including transverse coordinate ρ_R and atomic multipolar interaction strengths; thus the bar heights are for illustrative purposes only.

analysis can be extended to higher powers of ρ_q , leading to higher multipole excitations involving OAM transfer involving multiple units of \hbar . This situation is depicted in Fig. (1)(a).

For $l = 0$, we have $\mathcal{A}^{+1} = -\mathcal{A}^{-1*}$. Thus, besides the phase factor, the effective dichroic operator for a vortex beam interacting with an atom is directly comparable to the operator associated with the absorption and emission of a circularly polarised photon, $\hat{O}^\pm \sim (\hat{\epsilon}_x \pm i\hat{\epsilon}_y) \cdot \mathbf{r}_q = x_q \pm iy_q = \rho_q e^{\pm i\phi_q}$ [12]. The dichroic EV beam electron energy loss spectroscopy can be seen to be directly equivalent to the optical circularly polarised dichroic absorption spectroscopy by using an $l = 0$ incident Bessel beam, analysing the vorticity of the outgoing beams and comparing the intensity of the exiting $l' = \pm 1$ beams. Since the formal equivalence is at the operator level, the result applies to any quantum system with axial symmetry about \mathbf{R} . In this regard, vortex beam chiral spectroscopy is similar to EMCD [26] but would be much more practical because only small angle scattering is involved in the vortex beam case, so the signal-to-noise ratio should be much improved.

We can now address the physical meaning of the dou-

ble summation over p and p' in Eq. (20) and the implications of this for the chiral-specific spectroscopy of atoms located away from the vortex beam axis. It can be seen that the features uncovered above as regards OAM transfer from the EV to the atomic electron still apply, except that now the vortex states with which the atom interacts are characterised by the winding number p , not l , and the outgoing states after the electronic transition within the atom are characterised by p' rather than l' . This can be understood as a mode broadening effect of the EV beam arising from the translation of the reference axis, as described by Eq. (9) and Eq. (10). However, since chiral specific spectroscopy is normally conducted with respect to the beam axis, summed over atoms at various off-axis positions, the spectral changes observed in different OAM components of the EV beam can not be exactly related to the change in OAM of the atomic electronic system, as has been assumed in the case of [3]. The off-axis case is illustrated in Fig. (1)(b).

An exception is in the case of an atom whose centre of mass is in a pure OAM state, such as found in a Bose-Einstein condensate [27, 28]. In that case, the OAM states of the atoms would contribute a factor $e^{i(L-L')\phi_R}$ within the matrix element and we must integrate the factor F_R with respect to the dynamical variable ϕ_R .

$$\int_0^{2\pi} e^{i(l+L-p-l'-L'+p')\phi_R} d\phi_R \quad (23)$$

This gives rise to a selection rule for OAM transfer involving the atomic centre of mass such that $\delta p - \delta L = \delta l$. δp then corresponds to the net OAM change induced in the atomic system and so we recover the selection rules derived in [12, 17]. One way to understand this result is to view the azimuthally delocalized state of the atom interacting coherently with the vortex beam. It has been shown by Berry that the OAM of the beam is then purely intrinsic [24]. If we view the atom interacting with the EV beam as a detector, then the conservation of orbital angular momentum is dependent on the nature of the detector, and whether it has the cylindrical symmetry of the vortex beam. This argument could lead to a general test as to whether or not a system is coupled coherently to the EV.

In summary, we have carried out a comprehensive analysis of OAM transfer in inelastic atom-vortex interactions, and derived the effective operator exhibiting quantised OAM transfer via multipole electronic excitation of the atom. When the atom is on-axis, spectroscopy with the $l = 0$ beam is shown to be equivalent to magnetic circular dichroism, however, for the cases when the atom is off-axis, the extrinsic nature of the OAM of the vortex beam leads to a complicated situation in which the change in OAM of the vortex beam does not directly correlate with the change in OAM of the atom. As such, simple application of global OAM conservation is not the correct approach in analysing such interactions, and the

implication for chiral spectroscopy is that extreme care must be taken in experimental analysis, with the relative strengths for the multipole interactions taken into account. The dependence of the interaction on coherent coupling with the atomic state is noted, and this suggests a general test for coherence of the vortex interactions.

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- [1] K. Bliokh, Y. Bliokh, S. Savelev, and F. Nori, *Physical Review Letters* **99**, 190404 (2007).
 - [2] M. Uchida and A. Tonomura, *Nature* **464**, 737 (2010).
 - [3] J. Verbeeck, H. Tian, and P. Schattschneider, *Nature* **467**, 301 (2010).
 - [4] B. J. McMorran, A. Agrawal, I. M. Anderson, A. A. Herzog, H. J. Lezec, J. J. McClelland, and J. Unguris, *Science* (New York, N.Y.) **331**, 192 (2011).
 - [5] T. Gnanavel, J. Yuan, and M. Babiker, in *Proceedings of the European Microscopy Congress*, ii, edited by D. J. Stokes and J. Hutchison (Royal Microscopical Society, Oxford, 2012).
 - [6] P. Schattschneider, M. Stöger-Pollach, and J. Verbeeck, *Physical Review Letters* **109**, 084801 (2012).
 - [7] J. Verbeeck, H. Tian, and A. Béch , *Ultramicroscopy* **113**, 83 (2012).
 - [8] L. Allen, S. M. Barnett, and M. J. Padgett, *Optical Angular Momentum* (Institute of Physics Pub., Bristol, 2003).
 - [9] A. M. Yao and M. J. Padgett, *Advances in Optics and Photonics* **204**, 161 (2011).
 - [10] D. L. Andrews and M. Babiker, *The Angular Momentum of Light* (Cambridge University Press, Cambridge, 2012).
 - [11] M. Babiker, C. Bennett, D. Andrews, and L. D vila Romero, *Physical Review Letters* **89**, 1 (2002).
 - [12] S. M. Lloyd, M. Babiker, and J. Yuan, *Physical Review A* **86**, 023816 (2012).
 - [13] F. Araoka, T. Verbiest, K. Clays, and A. Persoons, *Physical Review A* **71**, 055401 (2005).
 - [14] W. L ffler, D. Broer, and J. Woerdman, *Physical Review A* **83**, 065801 (2011).
 - [15] N. B. Simpson, K. Dholakia, L. Allen, and M. J. Padgett, *Optics Letters* **22**, 52 (1997).
 - [16] G. Molina-Terriza, J. Torres, and L. Torner, *Physical Review Letters* **88**, 1 (2001).
 - [17] S. M. Lloyd, M. Babiker, and J. Yuan, *Physical Review Letters* **108**, 074802 (2012).
 - [18] S. J. Van Enk, *Quantum Optics: Journal of the European Optical* **6**, 445 (1994).
 - [19] M. Babiker, W. L. Power, and L. Allen, *Physical Review Letters* **73**, 1239 (1994).
 - [20] B. T. Thole, P. Carra, F. Sette, and G. Van der Laan, *Phys Rev Letts* **68**, 1943 (1992).
 - [21] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*, Applied mathematics series (Dover Publications, 1964).
 - [22] B. Korenev, *Bessel Functions and Their Applications*, Analytical Methods and Special Functions Series (Taylor & Francis, 2004).
 - [23] A. O’Neil, I. MacVicar, L. Allen, and M. Padgett, *Physical Review Letters* **88**, 5 (2002).
 - [24] M. V. Berry, in *Proceedings of SPIE International Conference on Singular Optics*, Vol. 3487, edited by S. M. Soskin (1998) pp. 1–5.
 - [25] J. Verbeeck, P. Schattschneider, S. Lazar, M. Stoger-Pollach, S. L ffler, A. Steiger-Thirsfeld, and G. Van Tendeloo, *Applied Physics Letters* **99**, 203109 (2011).
 - [26] P. Schattschneider, S. Rubino, C. H bert, J. Ruzs, J. Kunes, P. Nov k, E. Carlino, M. Fabrizio, G. Panaccione, and G. Rossi, *Nature* **441**, 486 (2006).
 - [27] M. Andersen, C. Ryu, P. Clad , V. Natarajan, A. Vaziri, K. Helmerson, and W. Phillips, *Physical Review Letters* **97**, 170406 (2006).
 - [28] A. Ramanathan, K. C. Wright, S. R. Muniz, M. Zelan, W. T. Hill, C. J. Lobb, K. Helmerson, W. D. Phillips, and G. K. Campbell, *Phys. Rev. Lett.* **106**, 130401 (2011).